 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

 **M.Sc.** DEGREE EXAMINATION - **STATISTICS**

SECOND SEMESTER – **APRIL 2012**

# ST 2902 – PROBABILITY THEORY AND STOCHASTIC PROCESSES

 Date : 23-04-2012 Dept. No. Max. : 100 Marks

 Time : 1:00 - 4:00

 **SECTION – A**

 **Answer All the questions. (10 x 2 = 20 Marks)**

1. Let A and B two events on the sample space. If A C B, show that P (A) < P (B)
2. If P (A∪B) = 0.7 , P(A) = 0.6 and P (B) = 0.5 , find P (A∩B) and P (Ac∩B)
3. Define conditional probability of events.
4. Define normal distribution.
5. Define a Renewal process.
6. If X has the following probability distribution:

 X = x: -3 -2 -1 0 1 2

P (X=x): 1/16 1/2 0 1/4 1/8 1/16

Find E (X).

1. Let x be a nonnegative random variable of the continuous type with pdf f and let α>0. If Y = Xα , find the pdf of Y.
2. Compute P (0 < X < 1/2 , 0 < Y < 1) if (X , Y) has the joint pdf

 f (x , y) = x2 + xy/3 , 0 < x <1 , 0 < y < 2

 0 , otherwise

1. Define communication of states of a Markov chain.
2. Write a note on Martingale.

**SECTION – B**

 **Answer any Five questions. (5 x 8 = 40 Marks)**

1. State and prove Boole’s inequality.
2. A problem in statistics was given to 3 students and whose probabilities of solving it are respectively 1/2 , 3/4 and 1/4 . What is the probability that (i) at least one will solve

(ii) exactly two will solve (iii) all the three will solve if they try independently.

1. If a random variable X has the pdf f (x) = 3x2 , 0 ≤ x < 1 , find a and b such that

(i) P (X ≤ a) = P (X >a) and (ii) P (X >b) = 0.05. Also compute P (1/4 < X < 1/2) .

 14. If X has pdf f (x) = k x2 e-x , 0 < x < ∞ , find (i) k (ii) mean (iii) variance

1. Let X be a standard normal variable. Find the pdf of Y = X2.
2. Explain the following (a) The Renewal function (b) Excess life (c) Current life

(d) Mean total life

1. If f (x, y) = 6 x2 y , 0 < x < 1 , 0 < y < 1, find (i) P (0 < X < 3/4 ∩ 1/3 < Y < 1/2)

(ii) (P (X < 1 | Y <2)

1. (a) Prove that communication is an equivalence relation.

(b) Write the three basic properties of period of a state.

**SECTION – C**

 **Answer any Two questions. (2 x 20 = 40 Marks)**

19. (a) State and prove Bayes’ theorem.

 (b) Consider 3 urns with the following composition of marbles.

 Urn Composition of Marbles

 White Red Black

 I 5 4 3

 II 4 6 5

 III 6 5 4

The probabilities of drawing the urns are respectively 1/5, ¼ and 11/20. One urn was chosen at random and 3 marbles were chosen from it. They were found to be 2 red 1 black. What is the probability that the chosen marbles would have come from urn I, urn II or urn III?

1. (a) If X has the probability mass function as P (X = x) = qxp ; x = 0, 1, 2, . . . . . ; 0 < p < 1 ,

 q = 1-p find the MGF of X and hence find mean and variance.

 (b) State and prove Lindeberg – Levy Central Limit Theorem.

1. Let f (x1 y) = 8xy , 0 < x < y <1 ; 0 , elsewhere be the joint pdf of X and Y. Find the conditional mean and variance of X given Y = y , 0 < y < 1 and Y given x = x , 0< x < 1.
2. (a) A Markov chain on states {0, 1, 2, 3, 4, 5} has transition probability matrix.

 1 0 0 0 0 0

 0 3/4 1/4 0 0 0

 0 1/8 7/8 0 0 0

1/4 1/4 0 1/8 3/8 0

1/3 0 1/6 1/6 1/3 0

 0 0 0 0 0 1

Find all equivalence classes and period of states. Also check for the recurrence of the

states.

 (b) Derive Yule process assuming that X (0) = 1 (10 +10)

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